

## Model Question Papers of

B.Sc Semester II

Mat C-4 - Integral Calculus &  
Analytical Geometry 3D.

Short answer type questions. (Integral Calculus)

1) Integrate

(a)  $\int (2x^2 + 3)\sqrt{x+4} \, dx.$

(b)  $\int (x+1)\sqrt{x^2+x+1} \, dx.$

(c)  $\int \frac{dx}{(x^2+4)\sqrt{x^2+9}}$

(d)  $\int \frac{dx}{(x-1)^2(x^2+4)}$

(e)  $\int \frac{x^2+1}{x(x^2-1)} \, dx.$

(f)  $\int \frac{dx}{x^4+1}$

② Evaluate from the first principle

(a)  $\int_0^{\pi/2} \cos x \, dx$

(b)  $\int_a^b x^2 \, dx$

③ Show that  $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} \, dx = \frac{\pi}{4}$

④ Show that  $\int_0^1 \frac{\log(1+x)}{1+x^2} \, dx = \frac{\pi}{8} \log 2$

⑤ Evaluate  $\int_0^{\pi/4} \tan^5 x \, dx$

⑥ Prove that  $\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx = \frac{8}{315}$

⑦ Trace the curve

(a)  $x^{2/3} + y^{2/3} = a^{2/3}$  (Astroid)

(b)  $y^2(a-x) = x^2(a+x)$

(c) Cardioid  $r = a(1 + \cos \theta)$

(d) curve  $r = a \sin 3\theta$

8) Find the area of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

9) Find the area of the cardioid

$$r = a(1 + \cos \theta)$$

10) Find the perimeter of the curve

$$x^{2/3} + y^{2/3} = a^{2/3}$$

### Short answer type questions (3-DIMENSION)

1) Prove that

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

where  $\alpha, \beta, \gamma$  are the angles made by the st. line with the  $x, y, z$ -axis respectively.

2) Prove that the equation of the plane in the intercept form is

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

3) Find the acute angle between the planes  $2x - y + z + 8 = 0$  and  $x + y + 2z - 14 = 0$ .

- ④ Express the general form of the equation of a line given by two first degree equations

$$a_1x + b_1y + c_1z + d_1 = 0,$$

$$a_2x + b_2y + c_2z + d_2 = 0.$$

in the symmetrical form.

- ⑤ Find the shortest distance between the lines

$$\frac{x-1}{2} = \frac{y+8}{-7} = \frac{z-4}{5} \quad \text{and}$$

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-6}{-3}$$

Long answer type questions  
(Integral Calculus)

① Evaluate  $\int_0^{\pi/2} \log \sin x \, dx$

② Evaluate the integral  $\int_0^{\pi/2} \sin^n x \, dx$

③  $\int_0^{\infty} x^n e^{-x} \, dx$

If  $n$  is a positive integer > 1, Prove that

DATE: \_\_\_\_\_

PAGE: \_\_\_\_\_

⑤

④ 
$$\int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx = \frac{1}{n-1}$$

⑤ 
$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} \, dx \quad \text{where } a > 1$$

⑥ Trace the curve  $y^2 = x(x-1)^2$  and find the area of its loop.

⑦ Find the whole area of the curve  $a^2 y^2 = x^2(a^2 - x^2)$

⑧ Find the area of the loop of the curve  $r^2 = a^2 \cos 2\theta$ .

⑨ Find the perimeter of the curve  $x^{2/3} + y^{2/3} = a^{2/3}$ .

⑩ Find the volume and surface area of a sphere of radius  $a$ .

## Long answer type Questions (Three Dimensions)

- ① If a line makes angles  $\alpha, \beta, \gamma, \delta$  with the diagonals of a cube, prove that

$$\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma + \sin^2 \delta = \frac{8}{3}$$

- ② Show that the lines whose direction cosines are given by  $l+m+n=0$  and  $2mn+3nl-5lm=0$  are perpendicular to each other.

- ③ Find the equation of the plane through the point  $(1, 1, 1)$  and perpendicular to the planes  $x-2y+z=2$  and  $4x+3y-z+1=0$ .

- ④ Find the length and the Equation of the Shortest Distance Between Two Skew Lines

$$\frac{x-a}{l} = \frac{y-b}{m} = \frac{z-c}{n}$$

$$\frac{x-a'}{l'} = \frac{y-b'}{m'} = \frac{z-c'}{n'}$$

- ⑤ Show that the sphere  $x^2+y^2+z^2+6y+2z+8=0$  and  $x^2+y^2+z^2+6x+8y+4z+20=0$  are orthogonal.